# Chapter 9

Right Triangles and Trigonometry

Section 1
Similar Right Triangles

GOAL 1: Proportions in Right Triangles

In Lesson 8.4, you learned that two triangles are similar if two of their corresponding angles are congruent. For example, $\triangle PQR \sim \triangle STU$. Recall that the corresponding side lengths of similar triangles are in proportion.

In the activity, you will see how a right triangle can be divided into two similar right triangles.
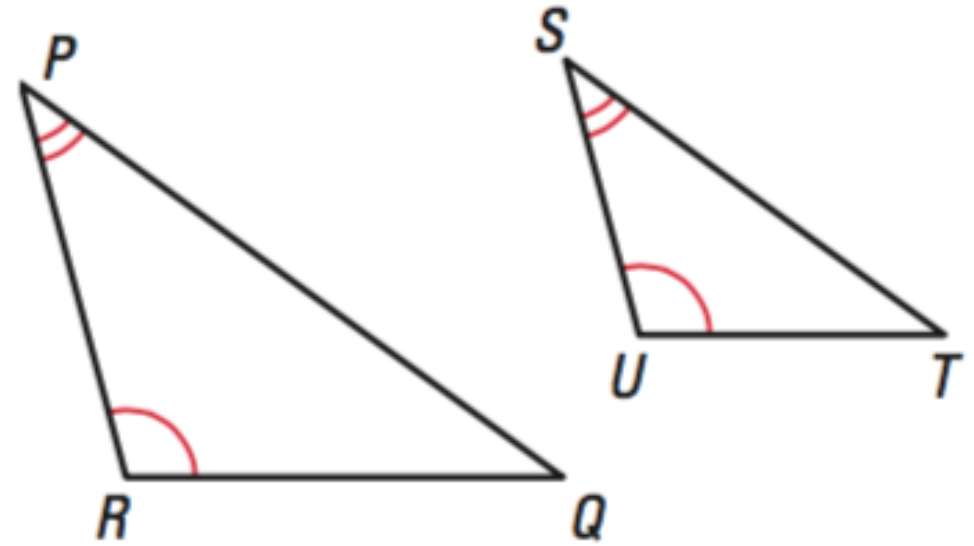
**Developing
Concepts**   **Investigating Similar Right Triangles**

1 Cut an index card along one of
its diagonals.

2 On one of the right triangles, draw
an altitude from the right angle to
the hypotenuse. Cut along the
altitude to form two right triangles.

3 You should now have three right
triangles. Compare the triangles.
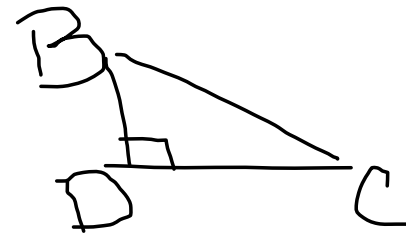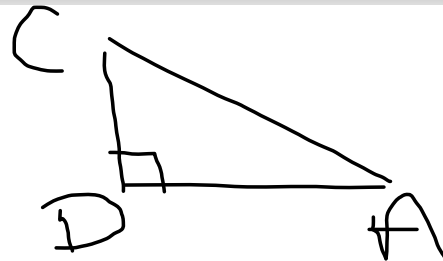What special property do they
share? Explain.
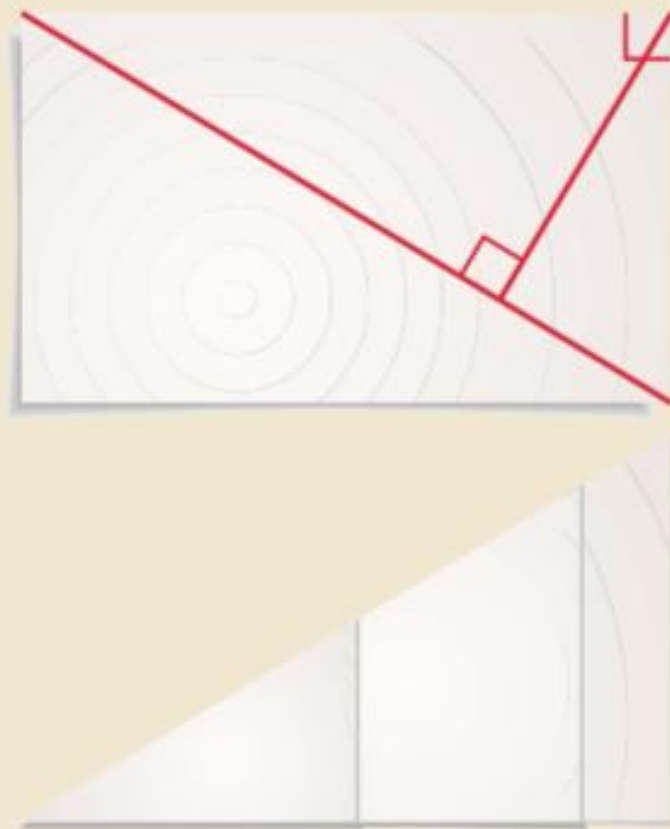
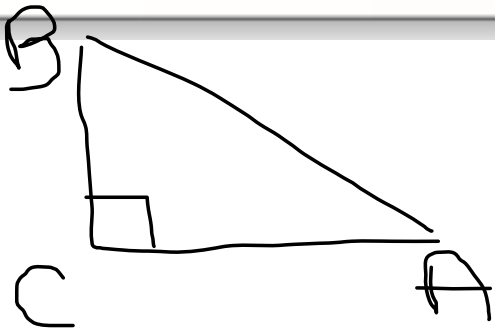## THEOREM 9.1

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.
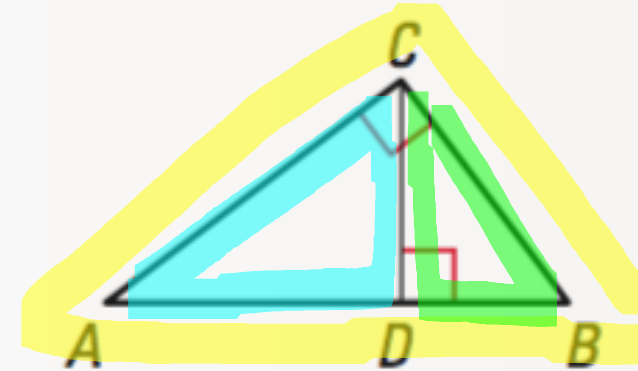
$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$

$$\triangle BCA \sim \triangle CDA \sim \triangle BDC$$

Example 1: Finding the Height of a Roof

A roof has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section.
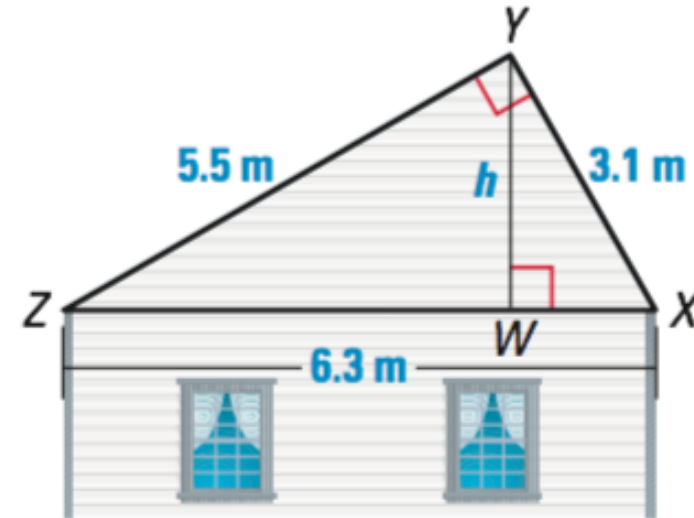
1) Identify the similar triangles.

$\triangle XYZ \sim \triangle YWZ \sim \triangle XWY$

2) Find the height h of the roof.

$$\frac{3.1}{6.3} = \frac{h}{5.5}$$
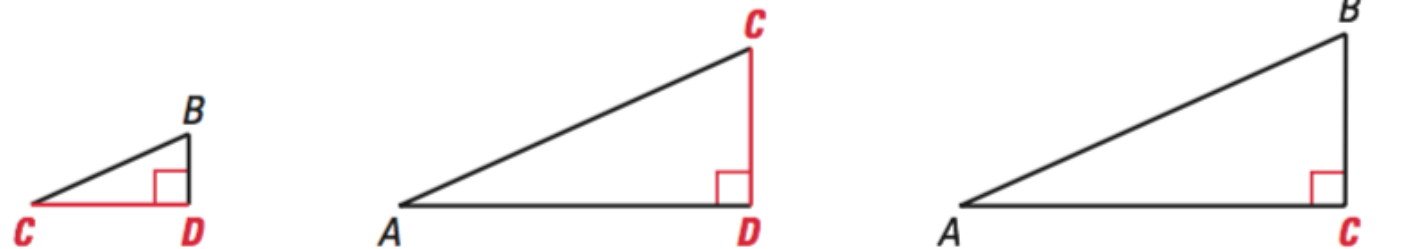
$$6.3h = 17.05$$

$$\frac{6.3h}{6.3} = \frac{17.05}{6.3}$$

$$\boxed{h = 2.7}$$

GOAL 2: Using a Geometric Mean to Solve Problems

In right $\triangle ABC$, altitude $\overline{CD}$ is drawn to the hypotenuse, forming two smaller right triangles that are similar to $\triangle ABC$. From Theorem 9.1, you know that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.

Notice that $\overline{CD}$ is the longer leg of $\triangle CBD$ and the shorter leg of $\triangle ACD$. When you write a proportion comparing the leg lengths of $\triangle CBD$ and $\triangle ACD$, you can see that CD is the *geometric mean* of BD and AD.

$$\text{shorter leg of } \triangle CBD \atop \text{shorter leg of } \triangle ACD \qquad \frac{BD}{CD} = \frac{CD}{AD} \qquad \text{longer leg of } \triangle CBD \atop \text{longer leg of } \triangle ACD$$

Sides $\overline{CB}$ and $\overline{AC}$ also appear in more than one triangle. Their side lengths are also geometric means, as shown by the proportions below:

$$\text{hypotenuse of } \triangle ABC \atop \text{hypotenuse of } \triangle CBD \qquad \frac{AB}{CB} = \frac{CB}{DB} \qquad \text{shorter leg of } \triangle ABC \atop \text{shorter leg of } \triangle CBD$$

$$\text{hypotenuse of } \triangle ABC \atop \text{hypotenuse of } \triangle ACD \qquad \frac{AB}{AC} = \frac{AC}{AD} \qquad \text{longer leg of } \triangle ABC \atop \text{longer leg of } \triangle ACD$$

GEOMETRIC MEAN THEOREMS

THEOREM 9.2

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

$$\frac{BD}{CD} = \frac{CD}{AD}$$
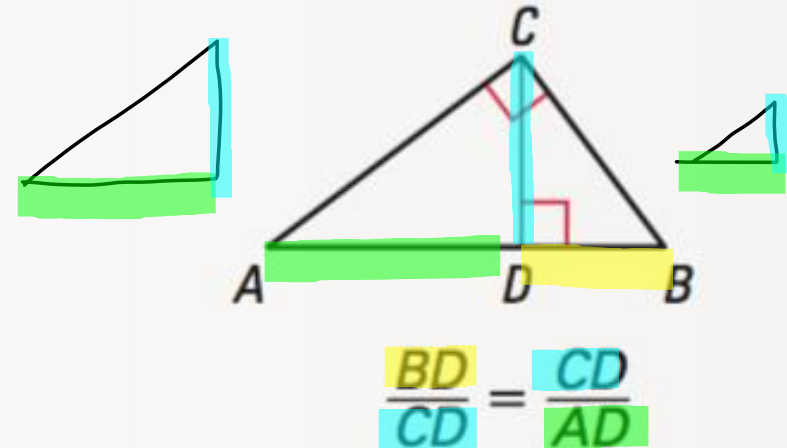
THEOREM 9.3

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.
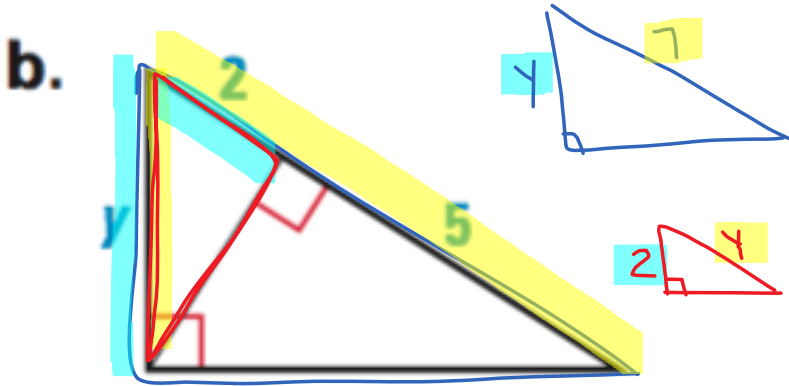
$$\frac{AB}{CB} = \frac{CB}{DB}$$

$$\frac{AB}{AC} = \frac{AC}{AD}$$

Example 2: Using a Geometric Mean

Find the value of each variable.

a.

$$\frac{6}{x} = \frac{x}{3}$$

$$\sqrt{x^2} = \sqrt{18}$$

$$x = 4.24$$

$$(3\sqrt{2})$$

b.
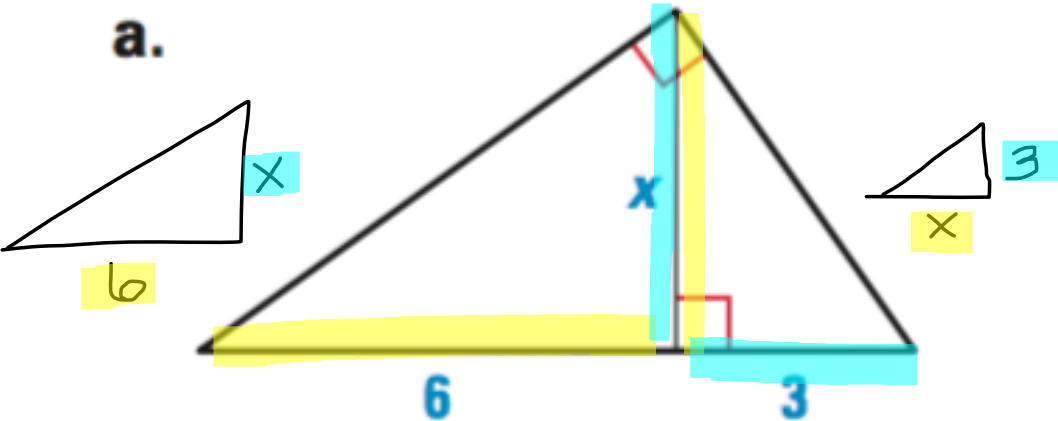
$$\frac{7}{y} = \frac{y}{2}$$

$$\sqrt{y^2} = \sqrt{14}$$

$$y = 3.7$$

Example 3: Using Indirect Measurement

MONORAIL TRACK To estimate the height of a monorail track, your friend holds a cardboard square at eye level. Your friend lines up the top edge of the square with the track and the bottom edge with the ground. You measure the distance from the ground to your friend's eye and the distance from your friend to the track.

In the diagram, $XY = h - 5.75$ is the difference between the track height h and your friend's eye level. Use Theorem 9.2 to write a proportion involving XY. Then you can solve for h.

Not drawn to scale

track

$h - 5.75$ ft

h

W 16 ft

5.75 ft
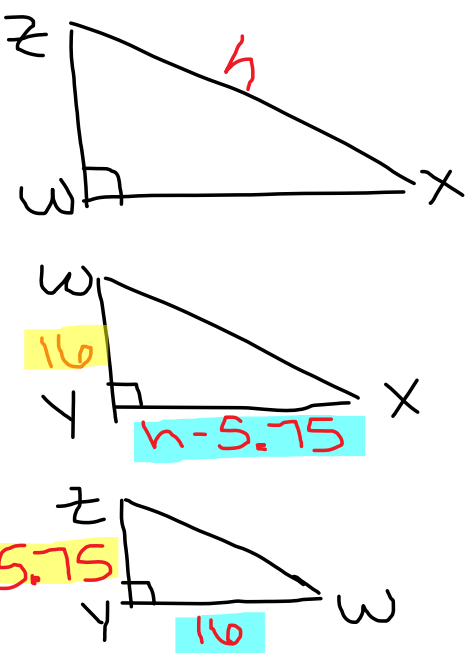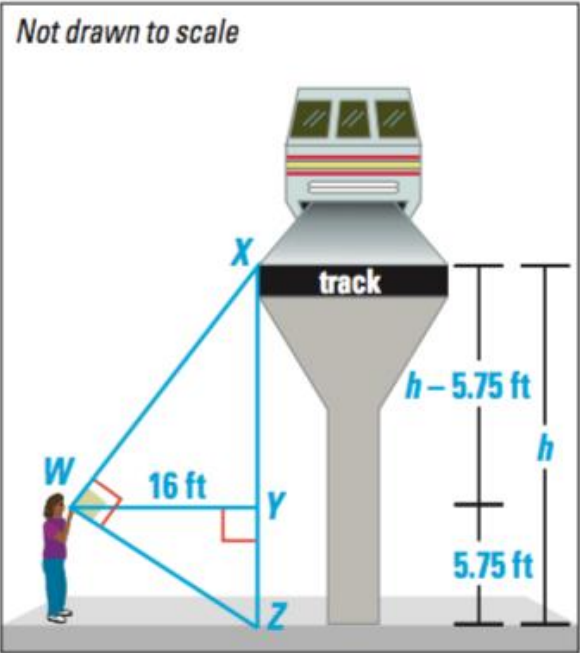
$$\frac{16}{5.75} \times \frac{h-5.75}{16}$$

$$5.75(h-5.75) = 256$$

$$5.75h - 33.0625 = 256$$
$$+33\ldots \qquad +33\ldots$$

$$\frac{5.75h}{5.75} = \frac{289.0625}{5.75}$$

$$h = 50.27 \text{ ft}$$

EXIT SLIP